Inequality

THE ATKINSON INDEX

Denote the Atkinson index of inequality for the group k by $I(k;\varepsilon)$. It can be expressed as follows:

$$\hat{I}(k;\varepsilon)=\frac{\hat{\mu}(k)-\hat{\xi}(k;\varepsilon)}{\hat{\mu}(k)} \quad \text{where} \quad \hat{\mu}(k)=\frac{\sum_{i=1}^{n} sw_i^k y_i}{\sum_{i=1}^{n} sw_i^k}$$

The Atkinson index of social welfare is as follows:

$$\hat{\xi}(k;\varepsilon)=\begin{cases}\left[\frac{1}{\sum_{i=1}^{n} sw_i^k}\sum_{i=1}^{n} sw_i^k (y_i)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} & \rightarrow \text{if } \varepsilon \neq 1 \text{ and } \varepsilon \geq 0 \\ \text{Exp}\left[\frac{1}{\sum_{i=1}^{n} sw_i^k}\sum_{i=1}^{n} sw_i^k \ln(y_i)\right] & \rightarrow \varepsilon = 1\end{cases}$$

If you wish to compute the Atkinson index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Atkinson index".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Atkinson index.
GRAPH: to draw the value of the index according to the parameter ρ.

S-GINI INDEX

Denoting the S-Gini index of inequality for the group k by $I(k;\rho)$, and the S-Gini social welfare index by $\xi(k;\rho)$, we have:

$$\hat{I}(k;\varepsilon)=\frac{\hat{\mu}(k)-\hat{\xi}(k;\rho)}{\hat{\mu}(k)}$$

where

$$\hat{\xi}(k;\rho) = \sum_{i=1}^{n}\left[\frac{(V_i)^{\rho}-(V_{i+1})^{\rho}}{[V_1]^{\rho}}\right]y_i \quad \text{and} \quad V_i = \sum_{h=i}^{n} sw_h^k \quad \text{and} \quad y_1 \geq y_2 \geq \cdots y_{n-1} \geq y_n \geq$$

REMARK: To compute the ordinary Gini index, the parameter ρ should be equals to 2.

If you wish to compute the S-Gini index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ S-Gini index".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Atkinson index.
GRAPH: to draw the value of the index according to the parameter ρ .

THE ATKINSON-GINI INDEX

Denoting the Atkinson-Gini index of inequality for the group k by $I(k;\varepsilon,\rho)$, and the Atkinson-S-Gini social welfare index by $\xi(k;\varepsilon,\rho)$, we have:

$$\hat{I}(k;\varepsilon,\rho) = \frac{\hat{\mu}(k)-\hat{\xi}(k;\varepsilon,\rho)}{\hat{\mu}(k)}$$

where

$$\hat{\xi}(k;\varepsilon,\rho) = \begin{cases} \left[\sum_{i=1}^{n}\left[\frac{(V_i)^{\rho}-(V_{i+1})^{\rho}}{(V_1)^{\rho}}\right](y_i)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \rightarrow \varepsilon \neq 1, \varepsilon \geq 0 \ \text{ and } \ \rho \geq 1 \\ \text{Exp}\left[\sum_{i=1}^{n}\left[\frac{(V_i)^{\rho}-(V_{i+1})^{\rho}}{(V_1)^{\rho}}\right]\ln(y_i)\right] \rightarrow \varepsilon = 1 \ \text{ and } \ \rho \geq 1 \end{cases}$$

and

$$V_i = \sum_{h=i}^{n} sw_h^k$$

and

$$y_1 \geq y_2 \geq \cdots y_{n-1} \geq y_n \geq$$

If you wish to compute the Atkinson-Gini index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Atkinson-Gini index".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Atkinson-Gini index.

THE GENERALISED ENTROPY INDEX

The Generalised Entropy Index of inequality for the group k is as follows:

$$\hat{I}(k;\theta)=\begin{cases} \dfrac{1}{\theta(\theta-1)\sum_{i=1}^{n} sw_i^k}\sum_i sw_i^k\left[\left(\dfrac{y_i}{\hat{\mu}(k)}\right)^{\theta}-1\right] & \text{if } \theta\neq 0,1 \\ \dfrac{1}{\sum_{i=1}^{n} sw_i^k}\sum_i sw_i^k \log\left(\dfrac{\hat{\mu}(k)}{y_i}\right) & \text{if } \theta=0 \\ \dfrac{1}{\sum_{i=1}^{n} sw_i^k}\sum_i \dfrac{sw_i^k y_i}{\hat{\mu}(k)}\log\left(\dfrac{y_i}{\hat{\mu}(k)}\right) & \text{if } \theta=1 \end{cases}$$

If you wish to compute the Generalised Entropy Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Entropy Index".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Generalised Entropy index.
GRAPH: to draw the value of the index according to the parameter θ.

THE QUANTILE RATIO AND THE INTERQUANTILE RATIO INDEX

Denote the Quantile Ratio for group k by $QR(k;p_1,p_2)$; it can be expressed as follows:

$$\widehat{QR}(k;p_1,p_2)=\frac{\hat{Q}(k,p_1)}{\hat{Q}(k,p_2)}$$

where $Q(k,p)$ denote the p-quantile of group k.

The Interquantile Ratio $IQR(k;p_1,p_2)$ is defined as:

$$\widehat{IQR}(k;p_1,p_2)=\frac{\hat{Q}(k,p_1)-\hat{Q}(k,p_2)}{\hat{\mu}(k)}$$

REMARK: The instructions for the Interquantile Ratio are similar to those for the Quantile Ratio.

If you wish to compute the Quantile Ratio Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Quantile Ratio".
- Choose the different vectors and values of parameters.

  Parameters

  p_1 : Percentile for numerator

  p_2 : Percentile for denominator

Among the buttons, you find the following commands:

COMPUTE: to compute the Quantile Ratio index.

THE COEFFICIENT OF VARIATION INDEX

Denote the Coefficient of Variation index of inequality for the group k by CV. It can be expressed as follows:

$$\widehat{CV}(k)=\left[\frac{\sum_{i=1}^{n} sw_i^k y_i^2 / \sum_{i=1}^{n} sw_i^k - \hat{\mu}(k)^2}{\hat{\mu}(k)^2}\right]^{\frac{1}{2}}$$

If you wish to compute the Coefficient of Variation Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Coefficient of Variation".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Coefficient of Variation index.

The Logarithmic Variance Index

Denote the Logarithmic Variance index of inequality for the group k by LV; it can be expressed as follows:

$$\widehat{LV}(k) = \frac{\sum_{i=1}^{n} sw_i^k \left(\log(y_i) - \log(\hat{\mu}(k))\right)^2}{\sum_{i=1}^{n} sw_i^k}$$

If you wish to compute the Logarithmic Variance Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Logarithmic Variance".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Logarithmic Variance index.

The Variance of Logarithms

Denote the Variance of Logarithms index of inequality for group k by VL. It can be expressed as follows:

$$\widehat{VL}(k) = \frac{\sum_{i=1}^{n} sw_i^k \left(\log(y_i) - \widehat{lmu}(k)\right)^2}{\sum_{i=1}^{n} sw_i^k} \quad \text{where} \quad \widehat{lmu}(k) = \frac{\sum_{i=1}^{n} sw_i^k \log(y_i)}{\sum_{i=1}^{n} sw_i^k}$$

If you wish to compute the Variance of Logarithms Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Variance of Logarithms".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Variance of Logarithms index.

The Relative Mean Deviation Index

Denote the Relative Mean Deviation index of inequality for the group k by RMD. It can be expressed as follows:

$$\widehat{\mathrm{RMD}}(k) = \frac{\sum_{i=1}^{n} sw_i^k \left| \left(y_i / \mu(k) \right) - 1 \right|}{\sum_{i=1}^{n} sw_i^k}$$

If you wish to compute the Relative Mean Deviation Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Relative Mean Deviation".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Relative Mean Deviation index.

The Conditional Mean Ratio

Denote the Conditional Mean for group k by $\mu(k;p_1;p_2)$, where p_1 and p_2 specify the percentile (p) range of those we wish to include in the computation of the conditional mean. These percentile values p are such that $p_1 \le p \le p_2$. $\mu(k;p_1;p_2)$ is formally defined as:

$$\hat{\mu}(k;\, p_1;p_2) = \frac{\int_{p1}^{p2} Q(k;p)dp}{p_2 - p_1}$$

and is the average income of those whose rank in the population is between p_1 and p_2. The Conditional Mean Ratio for group k is then given by CMR(k_1,k_2;,p_1,p_2,p_3,p_4) and is defined as

$$\mathrm{CMR}(k_1, k_2; p1, p2, p3, p4) = \frac{\mu(k_1;p_1;p_2)}{\mu(k_2;p_3;p_4)}$$

If you wish to compute the Conditional Mean Ratio Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Conditional Mean Ratio".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Conditional Mean Ratio index.

The Share Ratio

Denote the Share Ratio for population domain k by $SR(k; p1, p2, p3, p4)$, where p1 and p2 are lower and upper percentiles that delimitate a first group and p3 and p4 are lower and upper percentiles that delimitate a second group. The Share Ratio is the ratio of the income share of the first group over the income share of the second group:

$$\widehat{SR}(k;p1,p2,p3,p4) = \frac{\hat{L}(p2)-\hat{L}(p1)}{\hat{L}(p4)-\hat{L}(p3)}$$

If you wish to compute the Share Ratio Index of inequality, follow these steps:

- From the main menu, choose "Inequality⇒ Share Ratio".
- Choose the different vectors and values of parameters.

Among the buttons, you find the following commands:

COMPUTE: to compute the Share Ratio index.

Income-Component Proportional Growth

Change per 100 % Option

Let J components y^j add up to y, that is:

$$y_i = \sum_{j=1}^{J} y_i^j$$

The S-Gini index of inequality can be expressed as follows:

$$\hat{I}(\rho) = \sum_{j=1}^{J} \frac{\hat{\mu}_j}{\hat{\mu}_y} \widehat{IC}_j(\rho)$$

The contribution of the j^{th} component to total inequality in y is $\frac{\mu_j}{\mu_y} IC_j(\rho)$, where $IC_j(\rho)$ is the coefficient of concentration of the j^{th} component and μ_j is the mean of that component.

The impact on the S-Gini index of growth in y coming exclusively from growth in the j^{th} component is:

$$\frac{\frac{\partial I(\rho)}{\partial y^j}}{\frac{\partial \mu_y}{\partial y^j} / \mu_y} = \widehat{IC}_j(\rho) - \hat{I}(\rho)$$

When multiplied by 1%, this says for instance by how much (in absolute, not in percentage, terms) the Gini index will change if total income increases by 1% when that growth is entirely due to growth from the j^{th} component.
If you wish to compute this statistics, follow these steps:

- From the main menu, choose "Inequality⇒ Impact of Component Growth".
- Choose the different vectors and values of parameters.

Vectors

y : Variable of interest.

y^j : Component

Among the buttons, you find the following commands:

COMPUTE: to compute the impact on the S-Gini index of growth in y coming exclusively from growth in the j^{th} component.

ELASTICITY WITH RESPECT TO COMPONENT OPTION

The Gini j^{th} -component elasticity is given by:

$$\left(\frac{\frac{\partial I(\rho)}{\partial y^j}}{\frac{\partial \mu_y}{\partial y^j}}\right) / \left(\frac{I(\rho)}{\mu_y}\right) = \frac{\widehat{IC}_j(\rho)}{\hat{I}(\rho)} - 1$$

This gives the elasticity of the Gini index with respect to total income, when the change in total income is entirely due to growth from the j^{th} component.

If you wish to compute this statistics, follow these steps:

- From the main menu, choose "Inequality⇒ Gini Component Elasticity".
- Choose the different vectors and values of parameters.

Vectors

y : Variable of interest.

y^j : Component

Among the buttons, you find the following commands:

COMPUTE: to compute the Gini component elasticity.