# Poverty indices

DAD offers four possibilities for fixing the poverty line:

1- A deterministic poverty line set by the user.
2- A poverty line equal to a proportion "*l*" of the mean.
3- A poverty line equal to a proportion "*m*" of a quantile *Q(p)*.
4- An estimated poverty line that is asymptotically normally distributed with a standard deviation specified by the user.

For the first possibility, just indicate the value of the deterministic poverty line in front of "**Poverty line".** For the other three possibilities, proceed as follows:

- Click on the button "**Compute line**".
- Choose one of the following three options:

a) Proportion of mean: the proportion *l* should be entered.
b) Proportion of quantile: enter the proportion *m* and then the quantile *Q(p)* by specifying the desired percentile *p* of the population.
c) Estimated line: enter the estimate of the poverty line z and its standard deviation stdz.

### THE FGT INDEX

The unnormalized Foster-Greer-Thorbecke poverty index FGT P(k; z; α) for the population subgroup k is as follows:

$$P(k;z;\alpha)=\frac{1}{\sum_{i=1}^{n} sw_i^k}\sum_{i=1}^{n} sw_i^k (z-y_i)_+^{\alpha}$$

where z is the poverty line and $x_+ = \max(x,0)$. The normalized index is defined by:

$$\overline{P}(k;z;\alpha)=P(k;z;\alpha)/(z)^{\alpha}$$

If you wish to compute the FGT index of poverty, follow these steps:

- From the main menu, choose "Poverty⇒ FGT index".
- Choose the different vectors and values of parameters.

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the FGT index.
**GRAPH1:** to draw the value of the index according to the poverty line z.
**GRAPH2:** to draw the value of $FGT^{\frac{1}{\alpha}}$ as a function of a range of parameter $\alpha$

- To compute the normalized index, choose that option in the window of inputs.

## THE BOUNDED INCOME AND OVERLOAD INDICES

- **Gap index:**

The Gap index GI(k; z1; z2; α) for the population subgroup k is as follows:

$$GI(k;z1,z2;\alpha)=\frac{\sum_{i=1}^{n} sw_i^k (z2-y_i)^{\alpha} I(z1\le y_i\le z2)}{\sum_{i=1}^{n} sw_i^k}$$

If the index is relative to the group of those with $z1\le y_i\le z2$, we have:

$$GGI(k;z1,z2;\alpha)=\frac{\sum_{i=1}^{n} sw_i^k (z2-y_i)^{\alpha} I(z1\le y_i\le z2)}{\sum_{i=1}^{n} sw_i^k I(z1\le y_i\le z2)}$$

- **Surplus index:**

The Surplus index SI(k; z1; z2; α) for the population subgroup k is as follows:

$$SI(k;z1,z2;\alpha)=\frac{\sum_{i=1}^{n} sw_i^k (y_i-z1)^{\alpha} I(z1\le y_i\le z2)}{\sum_{i=1}^{n} sw_i^k}$$

If the index is relative to the group of those with $z1\le y_i\le z2$, we have:

$$GSI(k;z1,z2;\alpha)=\frac{\sum_{i=1}^{n} sw_i^k (y_i-z1)^{\alpha} I(z1\le y_i\le z2)}{\sum_{i=1}^{n} sw_i^k I(z1\le y_i\le z2)}$$

- **Overload index:**

The Overload Index $OLI(k,z,\alpha)$ for group k is as follows:

$$OLI(k,z,\alpha)=\frac{\widehat{GI}(k,z_1=0,z_2=z,\alpha)}{\widehat{SI}(k,z_1=z,z_2=+\infty,\alpha)}$$

If you wish to compute these indices of poverty, follow these steps:

- From the main menu, choose "Poverty⇒ Bounded income index ".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z1 | Lower bound | Compulsory |
| z2 | Upper bound | Compulsory |
| z | Poverty line | Compulsory for OLI |
| α | alpha | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the selected index.
**GRAPH:** to draw the value of the overload index as a function of a range of poverty lines z.

## THE WATTS POVERTY INDEX

The Watts poverty index $PW(k;z)$ for the population subgroup k is defined as:

$$PW(k;z) = -\frac{\sum_{i=1}^{n} sw_i^k \left(\log(y_i / z)\right)_+}{\sum_{i=1}^{n} sw_i^k}$$

where z is the poverty line and $x_+ = \max(x,0)$.

If you wish to compute the Watts index of poverty, follow these steps:

- From the main menu, choose "Poverty⇒ Watts index".
- Choose the different vectors and values of parameters.

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the Watts index.
**GRAPH:** to draw the value of the index according to the poverty line z.

## THE S-GINI POVERTY INDEX

The S-Gini poverty index $P(k;z;\rho)$ for the population subgroup k is defined as:

$$P(k;z;\rho) = z - \sum_{i=1}^{n}\left[\frac{(V_i)^\rho - (V_{i+1})^\rho}{[V_1]^\rho}\right](z - y_i)_+ \quad \text{and} \quad V_i = \sum_{h=i}^{n} sw_h^k$$

where z is the poverty line and $x_+ = \max(x,0)$.

If you wish to compute the S-Gini poverty index, follow these steps:

- From the main menu, choose "Poverty⇒ S-Gini index".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| ρ | Rho | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the S-Gini poverty index.
**GRAPH:** to draw the value of the index according to the poverty line z.

### THE CLARK, HEMMING AND ULPH (CHU) POVERTY INDEX

The poverty index $P(k;z;\varepsilon)$ for the population subgroup $k$ is defined as:

$$P(k;z,\varepsilon)=\begin{cases} z-\left(\dfrac{\sum_{i=1}^{n} sw_i^k (y_i^*)^{1-\varepsilon}}{\sum_{i=1}^{n} sw_i^k}\right)^{1/(1-\varepsilon)} & \text{if } \varepsilon\neq 1 \text{ and } \varepsilon\geq 0 \\ z-\exp\left(\dfrac{\sum_{i=1}^{n} sw_i^k \ln y_i^*}{\sum_{i=1}^{n} sw_i^k}\right) & \text{if } \varepsilon=1 \end{cases}$$

where z is the poverty line and $y_i^*=\begin{cases} y_i & \text{if } y_i\leq z \\ z & \text{otherwise} \end{cases}$

If you wish to compute the CHU poverty index of poverty, follow these steps:

- From the main menu, choose "Poverty⇒ CHU index".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the CHU poverty index.
**GRAPH:** to draw the value of the index according to the poverty line z.

## THE SEN INDEX

The Sen index of poverty $PS(k;z,\rho)$ for the population subgroup k is defined as:

$$PS = H\left[I + (1-I)G^{*}\right]$$

$$H = \frac{\sum_{i=i}^{n} sw_i^k * I(y_i^k \le z)}{\sum_{i=i}^{n} sw_i^k}$$

$$q = \frac{\sum_{i=i}^{n} sw_i^k * I(z - y_i^k)_+}{\sum_{i=i}^{n} sw_i^k}$$

$G^*$ is the Gini index of inequality among the poor, z is the poverty line and $x_+ = \max(x,0)$.

If you wish to compute the Sen poverty index, follow these steps:

- From the main menu, choose "Poverty⇒ Sen index".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the Sen poverty index.
**GRAPH:** to draw the value of the index according to the poverty line z.

## THE BI-DIMENSIONAL FGT INDEX

The Foster-Greer-Thorbecke poverty index, $P_g(k; z; \alpha)$, for an indicator of well-being g and for a population subgroup k is as follows

$$P_g(k;z_g;\alpha) = \frac{\sum_{i=1}^{n} sw_i^k (z_g - x_{g,i})_+^{\alpha}}{\sum_{i=1}^{n} sw_i^k}$$

where $z^g$ is the poverty line for good g, $x_{g,i}$ is the $g^{th}$ component of household I and $t_+ = \max(t,0)$. The normalised index is defined by:

$$\bar{P}_g(k;z_g;\alpha) = \bar{P}_g(k;z_g;\alpha)/(z_g)^{\alpha}$$

### Union headcount

The union headcount, based on G dimensions or commodities, is equal to:

$$P(k;z_1,z_2,...) = \frac{\sum_{i=1}^{n} sw_i^k \left(1 - \prod_{g=1}^{G} I(z_g < x_{g,i})\right)}{\sum_{i=1}^{n} sw_i^k}$$

### Intersection headcount

The intersection headcount, based on G dimensions or commodities, is equal to:

$$P(k;z_1,z_2,...) = \frac{\sum_{i=1}^{n} sw_i^k \prod_{g=1}^{G} I(z_g \geq x_{g,i})}{\sum_{i=1}^{n} sw_i^k}$$

### Union sum of gaps

The union sum of gaps, using G dimensions or commodities, is equal to:

$$P(k;z_1,z_2,...) = \frac{\sum_{i=1}^{n} sw_i^k \left(\sum_{g=1}^{G} (z_g - x_{g,i})_+\right)}{\sum_{i=1}^{n} sw_i^k}$$

### Intersection sum of gaps

The intersection sum of gaps, using G dimensions or commodities, is equal to:

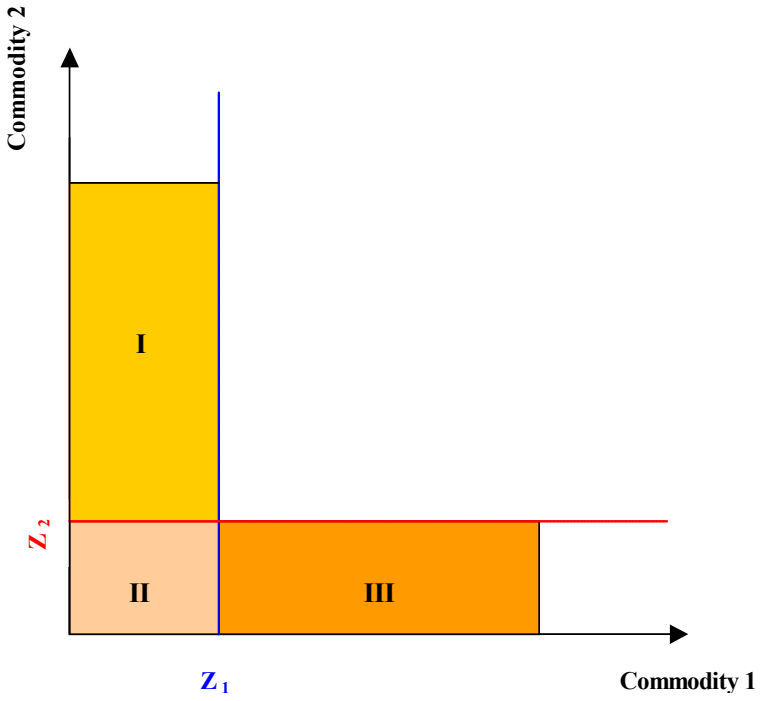
$$\hat{P}(k;z_1,z_2,...) = \frac{\sum_{i=1}^{n} sw_i^k \left(\sum_{g=1}^{G} (z_g - x_{g,i})_+ * \prod_{i=1}^{G} I(z_g \geq x_{g,i})\right)}{\sum_{i=1}^{n} sw_i^k}$$

### Intersection product of gaps

The intersection product of gaps, using G dimensions or commodities, is equal to:

$$P(k;z^1,z^2,...;\alpha_1,\alpha_2,...)=\frac{\sum_{i=1}^{n} sw_i^k\left(\prod_{g=1}^{G}(z_g-x_{g,i})^{\alpha_g}{}_{+}*\prod_{i=1}^{G}I(z_g\geq x_{g,i})\right)}{\sum_{i=1}^{n} sw_i^k}$$

**Graphical illustration for two commodities**




If you wish to compute bidimensional poverty indices, follow these steps:

- From the main menu, choose "Poverty⇒ Bidimensional index".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| $z_1$ | Poverty line 1 | Compulsory |
| $z_2$ | Poverty line 2 | Compulsory |
| $\alpha_1$ | alpha1 | Compulsory |
| $\alpha_2$ | alpha2 | Compulsory |

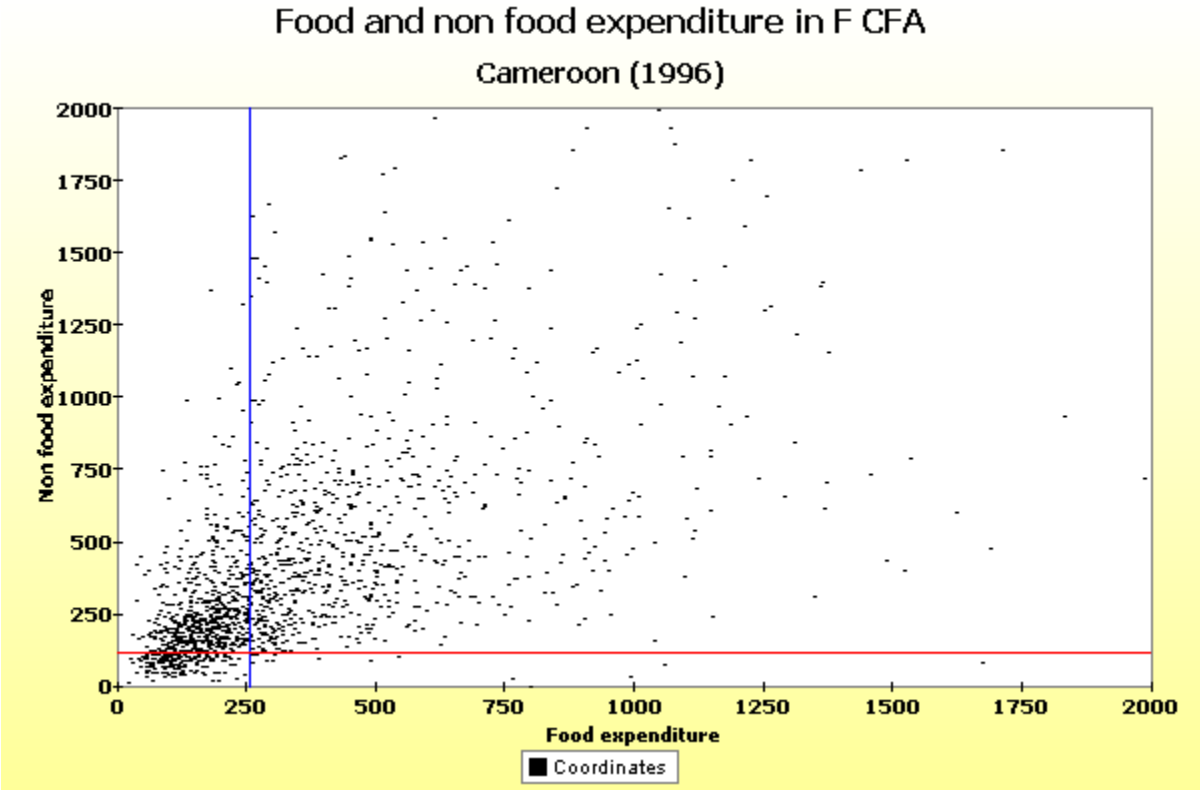
Among the buttons, you will find the following commands:

**COMPUTE:** to compute bidimensional poverty indices.

**Results of this application are:**

- FGT index for commodity 1: corresponding to areas (I+II) in the graphical illustration.
- FGT index for commodity 2: corresponding to areas (II+III) in the graphical illustration.
- FGT index for the two commodities (Union approach): corresponding to areas (I+II+III) in the graphical illustration.
- FGT index for the two commodities (Intersection approach): corresponding to areas (II) in the graphical illustration.

**Example:** Food and non-food expenditures per day in F CFA (Cameroon 1996). Food poverty line evaluated at 256 F CFA and non-food poverty line evaluated at 117 F CFA.




## IMPACT OF A PRICE CHANGE ON THE FGT INDEX

The impact of a good 1's marginal price change (denoted IMP) on the FGT poverty index $P(k,z;\alpha)$ is as follows:

$$IMP = \frac{\partial P(k;z;\alpha)}{\partial p_1} * pc$$

$$= \widehat{CD_1}^{\alpha+1}(k;z) * pc$$

where z is the poverty line, k is the population subgroup for which we wish to assess the impact of the price change, and pc is the percentage price change for good l.

$$IMP = \begin{cases} \dfrac{\alpha}{\sum_{i=1}^{n} sw_i^k z^\alpha} \sum_{i=1}^{n} sw_i^k \left( \dfrac{z - y_i}{z} \right)_+^{\alpha-1} x_i^1 & \text{if } \alpha \geq 1 \text{ and Normalised} \\ \dfrac{\alpha}{\sum_{i=1}^{n} sw_i^k} \sum_{i=1}^{n} sw_i^k (z - y_i)_+^{\alpha-1} x_i^1 & \text{if } \alpha \geq 1 \text{ and Not Normalised} \\ E\left[ x^1 \mid y = z \right] * f(z) = \dfrac{\sum_{i=1}^{n} sw_i^k K_h(z - y_i) * x_i^1}{\sum_{i=1}^{n} sw_i^k} & \text{if } \alpha = 0 \end{cases}$$

where $x_i^l$ is expenditure on commodity l by individual i, and $f_+ = \max(f, 0)$. Note that if the FGT index is normalized: $IMP = CD^{\alpha+1}{}_l(k; z) * pc$

If you wish to compute these statistics, follow these steps:

- From the main menu, choose "Poverty⇒ Impact of price change".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| α | Alpha | Compulsory |
| pc | Price change in % | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the impact of the price change.
**GRAPH:** to draw the value of the impact as a function of a range of poverty lines z.

## IMPACT OF A TAX REFORM ON THE FGT INDICES

A tax reform consists of a variation in the prices of two commodities 1 and 2, under the constraint that it leaves unchanged total government revenue. The effect of this constraint is given by an efficiency parameter, "gamma" ($\gamma$), which is the ratio of the marginal cost of public funds (MCPF) from a tax on 2 over the MCPF from a tax on 1.

The impact of this tax reform (denoted IMTR) on the FGT poverty index P(k; z; α) is given by:

$$IMTR = \left[ CD_1^{\alpha+1}(k;z) - \gamma \frac{\overline{X}_1}{\overline{X}_2} CD_2^{\alpha+1}(k;z) \right] * pc$$

where z is the poverty line, $CD_1^{\alpha+1}(k;z)$ and $CD_2^{\alpha+1}(k;z)$ are the consumption dominance curves for commodities 1 and 2, and pc is the percentage price change of commodity 1. Under the government revenue constraint, the percentage price change of commodity 1 is given by:

$$\gamma \frac{\overline{X}_1}{\overline{X}_2} pc.$$

To compute the impact of the tax reform:

- From the main menu, choose "Poverty⇒ Impact of tax reform".
- Choose the different vectors and values of parameters.

| | | |
|---|---|---|
| **Vectors** | | |
| $x_1$ | Commodity 1 | Compulsory |
| $x_2$ | Commodity 2 | Compulsory |
| **Parameters** | | |
| z | Poverty line | Compulsory |
| α | alpha | Compulsory |
| γ | gamma | Compulsory |
| pc | 1' s % price change | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE :** to compute the impact of the tax reform.

**CRITICAL γ :** to compute the gamma at which the tax reform will have zero impact on poverty. The value of this critical gamma equals: $\overline{CD}_1^{\alpha+1}(k;z) / \overline{CD}_2^{\alpha+1}(k;z)$.

**GRAPH z :** to draw the impact of the tax reform as a function of a range of poverty lines z.

**GRAPH $\delta_{1,2}$ :** **Graph γ ":** to draw the impact as a function of a range of MCPF ratios.

## LUMP-SUM TARGETING

The per-capita dollar impact of a marginal addition of a constant amount of income to everyone within a group k – called Lump-Sum Targeting (LST) – on the FGT poverty index P(k; z; α), is as follows:

$$LST = \begin{cases} -\alpha P(k,z;\alpha-1) \text{ if } \alpha \geq 1 \text{ and Not Normalised} \\ -\frac{\alpha}{z}\overline{P}(k,z;\alpha-1) \text{ if } \alpha \geq 1 \text{ and Normalised} \\ -f(k,z) \text{ if } \alpha = 0 \end{cases}$$

where z is the poverty line, k is the population subgroup for which we wish to assess the impact of the income change, and f(k,z) is the density function of the group k at level of income z.

To compute that impact:

- From the main menu, choose "Poverty⇒ Lump-sum Targeting".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| α | alpha | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the impact of the income change at a particular value of z
**GRAPH:** to draw the impact as a function of a range of poverty lines z.

### INEQUALITY-NEUTRAL TARGETING

The per-capita dollar impact of a proportional marginal variation of income within a group k, called Inequality Neutral Targeting, on the FGT poverty index P(k; z; α) is as follows:

$$INT = \begin{cases} \alpha \dfrac{P(k,z;\alpha) - zP(k,z;\alpha-1)}{\mu(k)} & \text{if } \alpha \geq 1 \text{ and FGT is not normalised} \\ \alpha \dfrac{\bar{P}(k,z;\alpha) - z\bar{P}(k,z;\alpha-1)}{\mu(k)} & \text{if } \alpha \geq 1 \text{ and FGT is normalised} \\ -\dfrac{zf(k,z)}{\mu(k)} & \text{if } \alpha = 0 \end{cases}$$

where z is the poverty line, k is the population subgroup for which we wish to assess the impact of the income change, and f(k,z) is the density function of the group k at level of income z.

To compute that impact:

- From the main menu, choose "Poverty⇒ Inequality-neutral Targeting ".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| α | alpha | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the impact of the income change.
**GRAPH:** to draw the impact as a function of a range of poverty lines z.

## FGT ELASTICITY

### Growth

The overall growth elasticity (GREL) of poverty, when growth comes exclusively from growth within a group k (namely, within that group, inequality neutral), is given by:

$$\text{GREL} = \begin{cases} \alpha \dfrac{P(k,z;\alpha) - zP(k,z;\alpha-1)}{P(z,\alpha)} & \text{if} \quad \alpha \geq 1 \\ -\dfrac{zf(k,z)}{F(z)} & \text{if} \quad \alpha = 0 \end{cases}$$

where z is the poverty line, k is the population subgroup in which growth takes place, f(z) is the density function at level of income z, and F(z) is the headcount.

### Gini

The overall Gini elasticity (GEL) of FGT poverty, is given by:

$$\text{GEL} = \begin{cases} \alpha\left(\dfrac{P(z;\alpha-1)}{P(z,\alpha)}\left(\dfrac{\mu}{z}-1\right)\right) & \text{if} \quad \alpha \geq 1 \\ \dfrac{f(z)(\mu - z)}{F(z)} & \text{if} \quad \alpha = 0 \end{cases}$$

To compute that growth elasticity:

- From the main menu, choose "Poverty⇒ Growth Elasticity".
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| α | Alpha | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the growth elasticity.
**GRAPH:** to draw the impact as a function of a range of poverty lines z.

## INCOME-COMPONENT PROPORTIONAL GROWTH

### Change per 100% of component

Assume that total income Y is the sum of C income components, with $Y = \sum_{c=1}^{C} \lambda_c y_c$ and where c is a factor that multiplies income component $y_c$ and that can be subject to growth. The derivative of the normalized FGT index with respect to $\lambda_c$ is given by

$$\left.\frac{\partial \bar{P}(k;z,\alpha)}{\partial \lambda_c}\right|_{\lambda_c=1, c=1\cdots C} = -CD_c(k;z,\alpha)$$

where $CD_c$ is the C-dominance curve of component c.

### Change per $ of component

The per-capita dollar impact of growth in the $j^{th}$ component on the normalized FGT index of the $k^{th}$ group is as follows:

$$\frac{\frac{\partial \bar{P}(k;z,\alpha)}{\partial y^j}}{\frac{\partial \mu(k)}{\partial y^j}} = -\overline{CD}^j(k;z,\alpha)$$

where $\overline{CD}$ is the normalized C-dominance curve of the component j.

### Elasticity with respect to component

The $j^{th}$ component elasticity of poverty (as measured by the FGT index) is:

$$-\frac{\mu(k)}{\bar{P}(k;z,\alpha)}\overline{CD}^j(k;z,\alpha)$$

where $\overline{CD}^j$ is the normalized C-dominance curve of the component j.

- If you wish to compute this elasticity, choose "Poverty⇒ Component Elasticity".
- If you wish to compute the above impacts, choose "Poverty⇒ Income-Component Proportional Growth", and select one of the three options.
- Choose the different vectors and values of parameters.

**Parameters**

| | | |
|---|---|---|
| z | Poverty line | Compulsory |
| $\alpha$ | Alpha | Compulsory |

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the statistics.

### THE IMPACT OF DEMOGRAPHIC CHANGES

This application computes the impact of a change (by a given percentage) in the population proportion of a group t. That change is accompanied by an exactly offsetting change in the population proportion of the other groups.

If the population proportion of group t increases by 100·pc percent, such that $\phi(t) \rightarrow (\phi(t)(1+pc))$, the total estimated impact on poverty is as follows:

$$\Delta P = \left( \phi(t) * P(t;z,\alpha) - \sum_{k \neq s}^{K} \frac{\phi(t)}{1-\phi(t)} * \phi(k) * P(k;z,\alpha) \right) * pc$$

If the population proportion of group s increases by an absolute 100·pc percent of the total population, such that $\phi(t) \rightarrow (\phi(t) + pc)$, the total estimated impact on poverty is as follows:

$$\Delta P = \left( P(k=t;z,\alpha) - \sum_{k \neq s}^{K} \frac{\phi(k)}{1-\phi(t)} * P(k;z,\alpha) \right) * pc$$

where $P(k;z;\alpha)$ is the FGT poverty index for subgroup k and $\phi(k)$ is the proportion of the population found in that subgroup.

To perform this estimation:

- From the main menu, choose: "Decomposition ⇒ Impact of Demographic Change".
- After setting the configuration, the application appears. Choose the different vectors and parameter values as follows:

**Parameters**

| | | |
|---|---|---|
| t | Changed group | Compulsory |
| z | Poverty line | Compulsory |
| $\alpha$ | Alpha | Compulsory |
| $k_1$-$k_2$-… | Group numbers separated by "-" | Compulsory |

**REMARK:** The group numbers separated by the dash "-" should be integer values. For example, we may have two subgroups coded by the integers 1 and 2. In this case, we would write in the field « **Group Numbers** » the values "1-2" before proceeding to the decomposition.