

The decomposition of inequality and poverty

THE DECOMPOSITION OF THE FGT INDEX

The FGT poverty index for a population composed of K groups can be written as

$$P(z; \alpha) = \sum_{k=1}^K \phi(k)P(k; z; \alpha)$$

where $P(k; z; \alpha)$ is the FGT poverty index for subgroup k and $\phi(k)$ is the proportion of the population in this subgroup. The contribution of group k to the poverty index for the whole population equals $\phi(k)P(k; z; \alpha)$.

To perform the decomposition of the FGT index:

- From the main menu, choose the item: "[Decomposition ⇒ FGT Decomposition](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Poverty line	z	Compulsory
alpha	α	Compulsory
Group numbers separated by "-"	$k_1 - k_2 - \dots$	Compulsory

REMARK: The group numbers separated by the dash "-" should be integer values. For example, we may have two subgroups coded by the integers 1 and 2. In this case, we would write in the field «**Group Numbers**» the values "1-2" before proceeding to the decomposition.

THE DECOMPOSITION OF THE FGT INDEX FOR TWO GROUPS

To perform the decomposition of the FGT index for two groups:

- From main menu, choose the item: "[Decomposition ⇒ FGT Decomposition for two groups](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values as follows

Parameters

Poverty line	z	Compulsory
alpha	α	Compulsory
Numbers for the 2 subgroups separated by "-"	$k_1 - k_2$	Compulsory

In the output window, you will find the following information:

- The FGT index for the whole population.
- The FGT index for each of the two subgroups.
- The difference in the indices of the two groups: $P(1; z; \alpha) - P(2; z; \alpha)$
- The percentage difference in the contribution of the two population subgroups, $(\phi(1)P(1; z; \alpha) - \phi(2)P(2; z; \alpha)) / P(z; \alpha)$

To compute the standard deviations for these statistics, choose the option computing with standard deviation.

THE DECOMPOSITION OF THE FGT INDEX ACROSS GROWTH AND REDISTRIBUTION EFFECTS

According to Datt & Ravallion (1992) approach, we can decompose variation of the FGT Index between two periods, t1 and t2, into growth and redistribution effects as follows:

$$\underbrace{P_2 - P_1}_{\text{variation}} = \underbrace{\left[P(\mu^{t2}, \pi^{t1}) - P(\mu^{t1}, \pi^{t1}) \right]}_{C1} + \underbrace{\left[P(\mu^{t1}, \pi^{t2}) - P(\mu^{t1}, \pi^{t1}) \right]}_{C2} + R \quad / \text{ref} = 1$$

$$\underbrace{P_2 - P_1}_{\text{variation}} = \underbrace{\left[P(\mu^{t2}, \pi^{t2}) - P(\mu^{t1}, \pi^{t2}) \right]}_{C1} + \underbrace{\left[P(\mu^{t2}, \pi^{t2}) - P(\mu^{t2}, \pi^{t1}) \right]}_{C2} + R \quad / \text{ref} = 2$$

Variation = Difference in poverty between t1 and t2.

C1 = Growth Impact.

C2 = Contribution of redistribution effect

R = Residual

Ref : Indicates the period of reference.

$P(\mu^{t2}, \pi^{t1})$: the FGT index of the first period when we multiply all incomes y_i^{t1} of the first period by the ratio μ^{t2} / μ^{t1}

$P(\mu^{t1}, \pi^{t2})$, the FGT index of the second period when we multiply all incomes y_i^{t2} of the second period by the ratio μ^{t1} / μ^{t2}

According the Shapley approach, we can decompose variation of the FGT Index between two periods, t1 and t2, into growth and redistribution effects as follows:

$$\underbrace{P_2 - P_1}_{\text{Variation}} = C_1 + C_2$$

$$C_1 = \frac{1}{2} \left(\left[P(\mu^{t2}, \pi^{t1}) - P(\mu^{t1}, \pi^{t1}) \right] + \left[P(\mu^{t2}, \pi^{t2}) - P(\mu^{t1}, \pi^{t2}) \right] \right)$$

$$C_2 = \frac{1}{2} \left(\left[P(\mu^{t1}, \pi^{t2}) - P(\mu^{t1}, \pi^{t1}) \right] + \left[P(\mu^{t2}, \pi^{t2}) - P(\mu^{t2}, \pi^{t1}) \right] \right)$$

To perform the decomposition of the FGT index across growth and redistribution effects:

- From the main menu, choose the item: "[Decomposition ⇒ Growth and redistribution](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Poverty line	z	Compulsory
alpha	α	Compulsory

We note here that the Shapley approach is equivalent to the axiomatic Kakwani approach for this decomposition.

THE SECTORAL DECOMPOSITION OF DIFFERENCES IN FGT INDICES

We can decompose differences in FGT into sub-group differences in poverty and population proportions as follows:

$$\underbrace{P_2 - P_1}_{\text{Variation}} = \left[\sum_{k=1}^K \underbrace{\phi_1(k) (P_2(k; z, \alpha) - P_1(k; z, \alpha))}_{C1} \right] + \left[\sum_{k=1}^K \underbrace{P_1(k; z, \alpha) (\phi_2(k) - \phi_1(k))}_{C2} \right] + \left[\sum_{k=1}^K \underbrace{(P_2(k; z, \alpha) - P_1(k; z, \alpha)) (\phi_2(k) - \phi_1(k))}_{C3} \right]$$

- Variation = Difference in poverty between 1 and 2.
- C1 = Intra-sectoral or intra-group impacts
- C2 = Impact of changes in subgroup proportions
- C3 = Interaction effect

To perform this decomposition:

- From the main menu, choose: "[Decomposition ⇒ Sectoral](#)".

- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Poverty line	z	Compulsory
alpha	α	Compulsory
Group numbers separated by "-"	$k_1 - k_2 - \dots$	Compulsory

We can always perform this decomposition by using the Shapley approach option such as:

$$\text{Contribution of } \phi_K = \frac{1}{2} \left[[(\phi_2(k)P_1(k; \alpha, z)) - (\phi_1(k)P_1(k; \alpha, z))] + [(\phi_2(k)P_2(k; \alpha, z)) - (\phi_1(k)P_2(k; \alpha, z))] \right]$$

$$\text{Contribution of } P_K = \frac{1}{2} \left[[(\phi_1(k)P_2(k; \alpha, z)) - (\phi_1(k)P_1(k; \alpha, z))] + [(\phi_2(k)P_2(k; \alpha, z)) - (\phi_2(k)P_1(k; \alpha, z))] \right]$$

THE DECOMPOSITION OF THE FGT INDEX BY SOURCES (OR COMPONENTS)

Let J components y_i^j add up to y_i , that is:

$$y_i = \sum_{j=1}^J y_i^j$$

The Shapley approach

One supposes with the Shapley approach that the contribution of component j towards reducing total poverty is the expected value of its marginal contribution when it is added randomly to anyone of the various subsets of components that one can choose from the set of all components.

When a component is missing from that set, we assume that the observation values of that component are everywhere replaced by 0.

To perform that decomposition of the FGT Index by sources:

- From the main menu, choose the item: " [Decomposition ⇒ FGT: Decomposition by sources](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

alpha	α	Compulsory
Poverty line	Z	Compulsory
Vector(s) of interest	To be selected	Compulsory

THE DECOMPOSITION OF THE FGT INDEX BY TRANSIENT & CHRONIC COMPONENTS

This type of decomposition decomposes total poverty, observed over some time periods, into transient and chronic components.

The Jalan & Ravallion approach

Let y_i^t be the income of household i in period t and μ_i be the average income over the T periods for household i . Total poverty is defined as follows:

$$TP(\alpha, z) = \frac{\sum_{t=1}^T \sum_{i=1}^N sw_i (z - y_i^t)_+^\alpha}{T \sum_{i=1}^N sw_i}$$

The chronic poverty component is then defined as:

$$CPC(\alpha, z) = \frac{\sum_{i=1}^N sw_i (z - \mu_i)_+^\alpha}{\sum_{i=1}^N sw_i}$$

The transient poverty component is finally defined as:

$$TPC(\alpha, z) = TP(\alpha, z) - CPC(\alpha, z)$$

The Duclos, Ximing and Araar approach

Let y_i^t be the income of household i in period t and μ_i be the average income over the T periods for household i . Let $\Gamma(\alpha, z)$ be the "equally-distributed-equivalent" (EDE) poverty gap such as:

$$\Gamma(\alpha, z) = [TP(\alpha, z)]^{1/\alpha} = \left[\frac{\sum_{t=1}^T \sum_{i=1}^N sw_i (z - y_i^t)_+^\alpha}{T \sum_{i=1}^N sw_i} \right]^{1/\alpha}$$

The transient poverty component is defined as follows:

$$TPC(\alpha, z) = \frac{\sum_{i=1}^N sw_i \theta_i(\alpha, z)}{\sum_{i=1}^N sw_i}$$

where $\theta_i = \gamma_i(\alpha, z) - \gamma_i(1, z)$ and $\gamma_i(\alpha, z) = \left(\left(\frac{\sum_{t=1}^T (z - y_i^t)_+^\alpha}{T} \right) \right)^{1/\alpha}$

The chronic poverty component is defined as follows:

$$CPC(\alpha, z) = \Gamma(\alpha, z) - TPC(\alpha, z)$$

Note that the number of periods available for this type of exercise is generally small. Because of this, a bias-correction is typically useful, using either an analytical/asymptotic or bootstrap approach.

To perform that decomposition of the FGT Index by transient and chronic components:

- From the main menu, choose the item: "[Decomposition ⇒ FGT: Transient and chronic](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

<u>Parameters</u>		
alpha	α	Compulsory
Poverty line	Z	Compulsory
Vector(s) of interest	To be selected	Compulsory

THE TRANSITION MATRIX

This type of decomposition gives us information on the final distribution (denoted by 1) for predetermined grouping within the initial distribution (denoted by 0).

Economic transition matrix

If we suppose that the bounds of the matrix are: $BR = \{0, B_1, \dots, B_i, B_j, \dots, B_{\max}\}$ the percentage of population:

- whose living standard is between B_j and B_k in the initial period
- whose living standard is between B_l and B_m in the final period

is given by:

$$\frac{\sum_{i=1}^N sw_i * I(B_j \leq y_i^0 < B_k) * I(B_l \leq y_i^1 < B_m)}{\sum_{i=1}^N sw_i}$$

Social transition matrix

The set-up is as above, except that the bounds are now estimated as quantiles of the initial and final distributions.

To perform such estimation:

- From the main menu, choose the item: "[Decomposition ⇒ Transient matrix](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

THE DECOMPOSITION OF THE S-GINI INDEX BY SOURCES (OR COMPONENTS)

Let J components y^j add up to y , that is:

$$y_i = \sum_{j=1}^J y_i^j$$

A “natural” approach

One “natural” approach to decomposing the S-Gini index of inequality is as follows:

$$I(\rho) = \sum_{j=1}^J \frac{\mu_j}{\mu} IC_j(\rho)$$

where $IC_j(\rho)$ is the coefficient of concentration of the j^{th} component and μ_j is the mean of that component. The contribution of the j^{th} component to inequality in y is then:

$$\frac{\mu_j}{\mu_y} IC_j(\rho),$$

The following results appear in the output window:

- The S-Gini index for y .
- The coefficients of concentration for every component of y .

- The ratio μ_j / μ for every component of y.
- The contribution for every component.

The Shapley approach

One supposes with the Shapley approach that the contribution of component j to total inequality is the expected value of its marginal contribution when it is added randomly to anyone of the various subsets of components that one can choose from the set of all components.

When a component is missing from that set, we assume that the observation values of that component are everywhere replaced by its average.

The following results appear in the output window:

To perform that decomposition of the S-Gini index of inequality:

- From the main menu, choose the item: "[Welfare and inequality ⇒ Decomposition ⇒ S-Gini decomposition](#)".
- Select the desired decomposition approach.
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Rho	ρ	Compulsory
Vector(s) of interest	Index1-index2...	Compulsory

THE DECOMPOSITION OF THE S-GINI INDEX BY POPULATION GROUPS

Let there be G population subgroups. We wish to determine the contribution of every one of those subgroups to total population inequality.

Natural approach

We rewrite the S-Gini index as:

$$I_\rho = \sum_{g=1}^G \phi_g^2 \frac{\mu_g}{\mu} I_{g,\rho} + \tilde{I}_\rho$$

where

- ϕ_g : the population share of group g;
- \tilde{I}_ρ : the contribution of inter group inequality to total inequality;
- μ_g : the average revenue of those in group g.
- μ : average revenue of total population.
- $I_{g,\rho}$: S-Gini of group g

The Shapley approach

This decomposition has two steps. The first one is to decompose total inequality into inter-group and intra-group contributions. The second step is to express the total intra-group contribution as a sum of the contributions of each of the groups.

In the first step, we suppose that the two Shapley factors are inter-group and intra-group inequality. The rules followed to compute inequality in the presence of one or two factors are:

- to eliminate intra-group inequality and to calculate inter-group inequality, we use a vector of incomes where each observation has the average income of its group;
- to eliminate inter-group inequality and to calculate intra-group inequality, we use a vector of incomes where each observation has its income multiplied by the ratio μ/μ_g .

The second step consists in decomposing total intra-group inequality as a sum of group inequality. To do this, we proceed systematically simply by replacing the revenues of those in a group by the average income of that group, such as to eliminate the intra-group contribution of a given group.

To perform the decomposition of the S-Gini index by groups:

- From the main menu, choose the item: "Welfare and inequality \Rightarrow [Decomposition](#) \Rightarrow [S-Gini decomposition by groups](#)".
- Select the desired decomposition approach.
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Rho	ρ	Compulsory
Vector(s) of interest	Index1-index2...	Compulsory

THE DECOMPOSITION OF THE GENERALISED ENTROPY INDEX OF INEQUALITY

The Generalised Entropy index of inequality can be decomposed as follows:

$$\hat{I}(\theta) = \sum_{k=1}^K \hat{\phi}(k) \left(\frac{\hat{\mu}(k)}{\hat{\mu}} \right)^\theta \cdot \hat{I}(k; \theta) + \hat{\bar{I}}(\theta)$$

where:

$\hat{\phi}(k)$ is the proportion of the population found in subgroup k.

$\hat{\mu}(k)$ is the mean income of group k.

$\hat{I}(k; \theta)$ is the inequality within group k.

$\hat{\bar{I}}(\theta)$ is population inequality if each individual in subgroup k is given the mean income of subgroup k, $\hat{\mu}(k)$.

To perform the decomposition of the entropy index:

- From the main menu, choose the item: "[Welfare and inequality](#) ⇒ [Decomposition](#) ⇒ [Entropy decomposition](#)".
- After confirming the configuration, the application appears. Choose the different vectors and parameter values.

Parameters

Theta	θ	Compulsory
Group numbers separated by "-"	$k_1 - k_2 - \dots$	Compulsory

The following information appears in the output window:

- The entropy index for the whole population.
- The entropy index for between-group inequality $\bar{I}(\theta)$.
- The entropy index within every subgroup $I(k; \theta)$.
- The ratio $(\mu(k)/\mu)$ "Normalised mean" for every subgroup.
- The absolute contribution to total inequality of inequality within every subgroup, that is, $(\mu(k)/\mu)^\theta \cdot \phi(k) \cdot I(k; \theta)$
- The relative contribution to total inequality of inequality within every subgroup.

[DECOMPOSITION OF VARIATION OF SOCIAL WELFARE INDEX BETWEEN TWO PERIODS](#)

We can decompose the difference in social welfare (as measured by the EDE Atkinson index) between two populations, 1 and 2, as follows:

$$\xi_2(\varepsilon) - \xi_1(\varepsilon) = \underbrace{(I_1 - I_2) * \mu_1}_{C1} + \underbrace{(\mu_2 - \mu_1) * (1 - I_1)}_{C2} + \underbrace{(\mu_2 - \mu_1) * (I_2 - I_1)}_{C3}$$

where:

- C1: Impact of change in inequality.
- C2: Impact of change in mean.
- C3: Interaction impact.

To perform this decomposition:

- From the main menu, choose: "Decomposition ⇒ [Decomposition of Social Welfare](#)".
- Choose the different vectors and parameter values.

Parameters

epsilon	ε_1	ε_2	Compulsory
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