

## Curves

A number of curves can be useful to present a general descriptive view of the distribution of living standards. Many of these curves can also serve to check the robustness of distributive orderings in terms of poverty, inequality, social welfare and equity.

### QUANTILES AND NORMALISED QUANTILES

**REMARK:** The application for computing normalised quantiles is similar in structure to the one for computing quantiles.

The  $p$ -quantile at a percentile  $p$  of a continuous population is given by:

$Q(p) = F^{-1}(p)$  where  $p = F(y)$  is the cumulative distribution function at  $y$ .

For a discrete distribution, let the  $n$  observations of living standards be ordered such that  $y_1 \leq y_2 \leq \dots \leq y_i \leq y_{i+1} \leq \dots \leq y_n$ . If  $F(y_i) < p \leq F(y_{i+1})$ , then we define  $Q(p) = y_{i+1}$ .

The normalised quantile is defined as  $\bar{Q}(p) = Q(p)/\mu$ .

To compute quantiles:

- From the main menu, choose the item: "**Curves  $\Rightarrow$  Quantile**".
- Choose the different vectors and parameter values.

#### Parameters

p: Percentile

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the quantile at a percentile  $p$ .

**GRAPH:** to draw the quantile curve according to the parameter  $p$

### Two distributions

Among the buttons, you will find the following commands:

**CROSSING:** to check if the two quantile curves intersect. If the two curves intersect, DAD indicates the co-ordinates of the first intersection and their standard deviation if the option of computing with standard deviation is chosen. To seek an intersection over a particular range of,  $p$  use **RANGE** to specify this range.

**GRAPH:** to draw the difference  $Q_1(p) - Q_2(p)$  along values of the parameter  $p$ .

**RANGE:** to specify the range for the search for a crossing of the two curves; this This also specifies the range of the horizontal axis over which a curve is drawn. when drawing a curve.

**DIFFERENCE:** to compute the difference  $Q_1(p_1) - Q_2(p_2)$ .

**POVERTY GAP CURVE**

The poverty gap at a percentile p is:

$$g(p; z) = (z - Q(p))_+$$

To compute a poverty gap:

- From the main menu, choose the item: "Curves  $\Rightarrow$  Poverty gap".
- Choose the different vectors and parameter values.

<u>Parameters</u>	
p:	Percentile

Among the buttons, you will find the following commands

**COMPUTE:** to compute  $g(p; z)$  at a percentile p.

**GRAPH:** to draw the value of  $g(p; z)$  as a function of p.

**Two distributions**

Among the buttons, you will find the following commands:

**CROSSING:** to search for the first intersection of the curves. If the two curves intersect, DAD indicates the co-ordinates of the first intersection and their standard deviation if the option of computing with standard deviation is chosen. To seek an intersection over a particular range, use **RANGE**.

**GRAPH:** : to draw the difference  $g_1(z_1; p) - g_2(z_1; p)$  as a function of p.

**RANGE:** to specify the range for the search for a crossing of the two curves. This also specifies the range of the horizontal axis over which a curve is drawn.

**DIFFERENCE:** to compute the difference  $g_1(z_1; p_1) - g_2(z_2; p_2)$ .

**LORENZ CURVE AND GENERALISED LORENZ CURVE**

The Lorenz curve at p for a population subgroup k is given by:

$$L(k; p) = \frac{\sum_{i=1}^n sw_i^k y_i I(y_i \leq Q(k; p))}{\sum_{i=1}^n sw_i^k y_i}$$

where  $I(y_i \leq Q(k; p)) = 1$  if  $y_i \leq Q(k; p)$  and 0 otherwise. and  $Q(k; p)$  is the p-quantile of the subgroup k.

The generalised Lorenz curve at p for a population subgroup k is:

$$GL(k;p) = \mu.L(k;p)$$

**REMARK:** The application for the Lorenz curve is similar in structure to the one for the generalised Lorenz curve.

To compute the Lorenz curve:

- From the main menu, choose the item: "Curves  $\Rightarrow$  Lorenz curve".
- Choose the different vectors and parameter values.

Parameters	
p:	Percentile

Among the buttons, you will find the following commands:

- COMPUTE:** to compute  $L(k;p)$ .
- GRAPH:** to draw the Lorenz curve
- RANGE:** to specify the range of the horizontal axis.

### Two distributions

Among the buttons, you will find the following commands:

- CROSSING:** to search for the first intersection of the curves. If the two curves intersect, DAD indicates the co-ordinates of the first intersection and their standard deviation if the option of computing with standard deviation is chosen. To seek an intersection over a particular range, use **RANGE**.
- GRAPH:** to draw the difference  $L_1(k_1;p) - L_2(k_2;p)$  as a function of p.
- RANGE:** to specify the range for the search for a crossing of the two curves. This also specifies the range of the horizontal axis over which a curve is drawn..
- DIFFERENCE:** to compute the difference:  $L_1(k_1;p_1) - L_1(k_2;p_2)$ .
- S-GINI:** to compute the difference  $I_1(k_1;\rho) - I_2(k_2;\rho)$ .
- COVARIANCE:** to compute the following covariance matrix
 

$\text{Cov}(L_1(k_1;0.1), L_2(k_2;0.1))$	$\text{Cov}(L_1(k_1;0.1), L_2(k_2;0.2))$	$\cdots$	$\text{Cov}(L_1(k_1;0.1), L_2(k_2;1))$
$\text{Cov}(L_1(k_1;0.2), L_2(k_2;0.1))$	$\text{Cov}(L_1(k_1;0.2), L_2(k_2;0.2))$	$\cdots$	
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\text{Cov}(L_1(k_1;1), L_2(k_2;0.1))$	$\text{Cov}(L_1(k_1;1), L_2(k_2;0.2))$	$\cdots$	$\text{Cov}(L_1(k_1;1), L_2(k_2;1))$

## CONCENTRATION CURVE AND GENERALISED CONCENTRATION CURVE

The concentration curve for the variable T ordered in terms of y at percentile p and for a population subgroup k is:

$$C_T(k;p) = \frac{\sum_{i=1}^n sw_i^k T_i I(y_i \leq Q(k;p))}{\sum_{i=1}^n sw_i^k T_i}$$

where  $I(y_i \leq Q(k;p)) = 1$  if  $y_i \leq Q(k;p)$  and 0 otherwise, and  $Q(k;p)$  is the p-quantile of y for the subgroup k.

The generalised concentration curve at p for a population subgroup p is:

$$C_T(k;p) = \frac{\sum_{i=1}^n sw_i^k T_i I(y_i \leq Q(k;p))}{\sum_{i=1}^n sw_i^k}$$

**REMARK:** The application for the generalized concentration curve is similar in structure to the one for the concentration curve.

To compute the concentration curve:

- From the main menu, choose the item: "[Curves ⇒ concentration curve](#)".
- Choose the different vectors and parameter values.

Parameters	
p:	Percentile

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the concentration curve  $C(k;p)$ .

**GRAPH:** to draw the concentration curve.

**RANGE:** to specify the range of the horizontal axis.

### Two distributions

Among the buttons, you will find the following commands:

**CROSSING:** to search for the first intersection of the curves. If the two curves intersect, DAD indicates the co-ordinates of the first intersection and their standard deviation if the option of computing with standard deviation is chosen. To seek an intersection over a particular range, use **RANGE**.

**GRAPH:** to draw the difference  $C_1(k_1;p) - C_2(k_2;p)$  as a function of p.

**RANGE:** to specify the range for the search for a crossing of the two curves. This also specifies the range of the horizontal axis over which a curve is drawn.

**DIFFERENCE:** to compute the difference:  $C_1(k_1;p) - C_2(k_2;p)$ .

**S-GINI:** to compute the difference  $IC_1(k_1; \rho) - IC_2(k_2; \rho)$ .

**COVARIANCE:** to compute the following covariance matrix

$Cov(C_1(k_1; 0.1), C_2(k_2; 0.1))$	$Cov(C_1(k_1; 0.1), C_2(k_2; 0.2))$	...	$Cov(C_1(k_1; 0.1), C_2(k_2; 1))$
$Cov(C_1(k_1; 0.2), C_2(k_2; 0.1))$	$Cov(C_1(k_1; 0.2), C_2(k_2; 0.2))$	...	$\vdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$Cov(C_1(k_1; 1), C_2(k_2; 0.1))$	$Cov(C_1(k_1; 1), C_2(k_2; 0.2))$	...	$Cov(C_1(k_1; 1), C_2(k_2; 1))$

**BIPOLARISATION CURVE**

**First order s = 1;**

The first order bipolarisation curve at p for a population subgroup k is given by:

$$BP(k; p, s = 1) = \frac{|Q(k, p) - Q(k, 0.5)|}{Q(k, 0.5)}$$

where  $Q(k; p)$  is the p-quantile of the subgroup k.

**First order s = 2;**

The first second bipolarisation curve at p for a population subgroup k is given by:

$$BP(k; p, s = 2) = 0.5 - p + \frac{\mu(k)}{Q(k, 0.5)} [L(p) - L(0.5)]$$

To compute the bipolarisation curves:

- From the main menu, choose the item: "[Curves ⇒ Bipolarisation curve](#)".
- Choose the different vectors and parameter values.

Parameters	
p:	The percentile
s:	The order s

Among the buttons, you will find the following commands:

<b>COMPUTE:</b>	to compute $BP(k; p, s)$ .
<b>GRAPH:</b>	to draw the bipolarisation curve.
<b>RANGE:</b>	to specify the range of the horizontal axis.

**Two distributions**

Among the buttons, you will find the following commands:

<b>GRAPH:</b>	to draw the difference $BP_1(k_1; p, s) - BP_2(k_2; p, s)$ as a function of p.
<b>RANGE:</b>	to specify the range for the search for a crossing of the two curves. This also

	specifies the range of the horizontal axis over which a curve is drawn..
<b>DIFFERENCE:</b>	to compute the difference: $BP_1(k_1; p, s) - BP_2(k_2; p, s)$ .

### THE CUMULATIVE POVERTY GAP (CPG) CURVE

The CPG curve at p for a subgroup k and a poverty line z is:

$$G(k; p; z) = \frac{\sum_{i=1}^n sw_i^k (z - y_i)_+ I(y_i \leq Q(k; p))}{\sum_{i=1}^n sw_i^k}$$

To compute the CPG curve for one distribution:

- From the main menu, choose the item: "Curves  $\Rightarrow$  CPG curve".
- Choose the different vectors and parameter values.

#### Parameters

- p: Percentile
- z: Poverty line

Among the buttons, you will find the following commands:

**COMPUTE:** to compute the CPG curve  $G(k; p; z)$ .

**GRAPH:** to draw the curve as a function of p.

**RANGE:** to specify the range of the horizontal axis.

### Two distributions

Among the buttons, you will find the following commands:

**CROSSING:** to search for the first intersection of the curves. If the two curves intersect, DAD indicates the co-ordinates of the first intersection and their standard deviation if the option of computing with standard deviation is chosen. To seek an intersection over a particular range, use **RANGE**.

**GRAPH:** to draw the difference  $G_1(k_1; p; z) - G_2(k_2; p; z)$  as a function of p.

**RANGE:** to specify the range for the search for a crossing of the two curves. This also specifies the range of the horizontal axis over which a curve is drawn..

**DIFFERENCE:** to compute the difference:  $G_1(k_1; p_1; z) - G_2(k_2; p_2; z)$ .

**S-GINI:** to compute the difference  $P_1(z_1; \rho) - P_2(z_1; \rho)$ .

**COVARIANCE:** to compute the following covariance matrix

$$\begin{matrix} \text{Cov}(G_1(k_1; 0.1; z_1), G_2(k_2; 0.1; z_2)) & \text{Cov}(G_1(k_1; 0.1; z_1), G_2(k_2; 0.2; z_2)) & \cdots & \text{Cov}(G_1(k_1; 0.1; z_1), G_2(k_2; l; z_2)) \\ \text{Cov}(G_1(k_1; 0.2; z_1), G_2(k_2; 0.1; z_2)) & \text{Cov}(G_1(k_1; 0.2; z_1), G_2(k_2; 0.2; z_2)) & \cdots & \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(G_1(k_1; l; z_1), G_2(k_2; 0.1; z_2)) & \text{Cov}(G_1(k_1; l; z_1), G_2(k_2; 0.2; z_2)) & \cdots & \text{Cov}(G_1(k_1; l; z_1), G_2(k_2; l; z_2)) \end{matrix}$$

### C-DOMINANCE CURVE

The  $j^{\text{th}}$  Commodity or Component dominance (C-Dominance for short) curve is defined as follows:

$$CD^j(k; z, s) = \begin{cases} (s-1) \frac{1}{\sum_{i=1}^n sw_i^k} \sum_{i=1}^n sw_i^k (z - y_i)_+^{s-2} y_i^j & \text{if } s \geq 2 \\ E[y^j | y = z] f(z) = \frac{\sum_{i=1}^n sw_i^k K(z - y_i) y_i^j}{\sum_{i=1}^n sw_i^k} & \text{if } s = 1 \end{cases}$$

where  $K(\cdot)$  is a kernel function and  $y^j$  is the  $j^{\text{th}}$  commodity. Dominance of order  $s$  is checked by setting  $\alpha = s - 1$ .

The C-Dominance curve normalized by  $z$ , which is denoted by  $\overline{CD}$ , is given by:

$$\overline{CD}^j(k; z, s) = \begin{cases} \frac{(s-1)}{z^{s-1}} \frac{1}{\sum_{i=1}^n sw_i^k} \sum_{i=1}^n sw_i^k (z - y_i)_+^{s-2} y_i^j & \text{if } s \geq 2 \\ E[y^j | y = z] f(z) = \frac{\sum_{i=1}^n sw_i^k K(z - y_i) y_i^j}{\sum_{i=1}^n sw_i^k} & \text{if } s = 1 \end{cases}$$

The C-Dominance curve normalized by the mean is defined as  $\frac{CD^j}{\mu^j}$  (where  $\mu^j$  is the mean of the  $j^{\text{th}}$  commodity), and the C-Dominance curve normalized both by  $z$  and the mean equals:

$$\frac{\overline{CD}^j}{z}$$

To compute the C-Dominance curve:

- From the main menu, choose: "**Curves  $\Rightarrow$  C-Dominance curve**".
- Choose the different vectors and parameter values.

**Parameters**

- p: Percentile
- s: Order  $s$

Among the buttons, you will find:

**COMPUTE:** to compute the C-Dominance curve at  $z$  and for a given  $s$ .

**GRAPH:** to draw the C-Dominance curve over a range of  $z$ .

**RANGE:** to specify the range of the horizontal axis.

### Two distributions

Among the buttons, you will find the following commands:

**DIFFERENCE:** compute the difference:  $CD^{1,j}(k; z, s) - CD^{2,j}(k; z, s)$

**GRAPH:** to draw the difference  $CD^{1,j}(k; z, s) - CD^{2,j}(k; z, s)$  as a function of  $z$ .

**RANGE:** to specify the range of the horizontal axis.

Where  $CD^{i,j}(k; z, s)$  is the C-Dominance of  $j^{\text{th}}$  component for distribution  $i$ .

### THE RELATIVE DEPRIVATION CURVE

Let the relative deprivation of an individual with income  $Q(p)$ , when comparing himself to another individual with income  $Q(q)$ , be given by:

$$\delta(q, p) = \begin{cases} 0, & \text{if } Q(p) \geq Q(q) \\ Q(q) - Q(p) & \text{Otherwise} \end{cases}$$

$$\bar{\delta}(p) = \int_0^1 \delta(q, p) dq$$

To compute the relative deprivation curve along values of percentiles  $p$ :

- From the main menu, choose the item: "[Curves  \$\Rightarrow\$  Relative Deprivation curve](#)".
- Choose the different vectors and parameter values.

#### Parameters

$p$ : Percentile

Among the buttons, you will find the following commands:

**COMPUTE:** to compute  $\bar{\delta}(p)$ .

**GRAPH:** to draw the curve as a function of  $p$ .

**RANGE:** to specify the range of the horizontal axis.

### Two distributions.

Among the buttons, you will find the following commands:

**DIFFERENCE:** compute the difference:  $\bar{\delta}_1(k_1, p_1) - \bar{\delta}_2(k_2, p_2)$

**GRAPH:** to draw the difference  $\bar{\delta}_1(k_1, p) - \bar{\delta}_2(k_2, p)$  as a function of  $p$ .



**RANGE:** to specify the range of the horizontal axis.

## PRO POOR CURVE

- **Income growth curve** : % change in  $Q(p)$  between two periods (1,2)

$$IGC(p) = \frac{Q_1(p) - Q_2(p)}{Q_1(p)}$$

- **Cumulative income growth curve** : % change in  $GL(p)$  between two periods (1,2)

$$CIGC(p) = \frac{GL_1(p) - GL_2(p)}{GL_1(p)}$$

If we note the growth standard between the two periods by  $\gamma$ , the growth is pro-poor if:

$$IGC(p) > \gamma \quad \forall p \in [0,1] \quad \text{for } s = 1 \text{ (first order)}$$

$$CIGC(p) > \gamma \quad \forall p \in [0,1] \quad \text{for } s = 2 \text{ (second order)}$$

- If  $CIGC(p) > \gamma$  for all  $p$  and  $\gamma = (\mu_1 - \mu_2) / \mu_1$  then growth is pro-poor because it shifts the Lorenz curve upward.
- If  $\gamma > CIGC(p) > 0$  for all  $p$ , then growth reduces absolute poverty but is accompanied by an increase in inequality ( $\Delta L(p) < 0$  for all  $p$ ). This situation may be characterized as trickle-down growth; growth reduces poverty but the poor receive proportionally less benefits than the non-poor.
- If  $CIGC(p) < 0$  for all  $p$  and  $g$  is positive, then we have that growth increases poverty

To compute the pro-poor curves:

- From the main menu, choose the item: "[Curves  \$\Rightarrow\$  Pro poor curve](#)".
- Chose the order
- Choose the different vectors and parameter values.

### Parameters

p: Percentile

Among the buttons, you will find the following commands:

- COMPUTE:** to compute the curve at percentile  $p$ .
- GRAPH:** to draw the curve as a function of  $p$ .
- RANGE:** to specify the range of the horizontal axis.