Redistribution

ESTIMATING THE PROGRESSIVITY OF A TAX OR A TRANSFER

Let:
- X be gross income;
- T be a tax;
- B be a transfer.

TR progressivity:

A tax T is TR-progressive if : $L_X(p) - C_T(p) > 0 \quad \forall p \in]0,1[$

A transfer B is TR-progressive if : $C_B(p) - L_X(p) > 0 \quad \forall p \in]0,1[$

IR-progressivity:

A tax T is IR-progressive if : $C_{X-T}(p) - L_X(p) > 0 \quad \forall p \in]0,1[$

A transfer B is IR-progressive if : $C_{X+B}(p) - L_X(p) > 0 \quad \forall p \in]0,1[$

To reach this application:

- From the main menu, choose the item: "Redistribution ⇒ Tax or transfer".
- Specify if you wish to estimate the progressivity of a tax or of a transfer.
- Choose the approach to be either TR or IR.
- Choose the different vectors and parameter values.

Parameters

p: Percentilr

ρ : Rho

Among the buttons, you find the following commands:

S-GINI: to compute as follows:

	TR Approach	IR Approach
Tax	$IC_T(\rho) - I_X(\rho)$	$I_X(\rho) - IC_{X-T}(\rho)$
Transfer	$I_X(\rho) - IC_B(\rho)$	$I_X(\rho) - IC_{X+B}(\rho)$

where $IC(\rho)$ is the S-Gini coefficient of concentration and $I(\rho)$ is the S-Gini index of inequality

CROSSING: to seek the first intersection of the concentration and Lorenz curves. DAD indicates the co-ordinates of that first intersection and their

standard deviation if the option of computing with standard deviation is chosen

DIFERENCE: to compute as follows:

	TR Approach	IR Approach
Tax	$L_X(p) - C_T(p)$	$C_{X-T}(p) - L_X(p)$
Transfer	$C_B(p) - L_X(p)$	$C_{X+B}(p) - L_X(p)$

GRAPH: to compute as follows:

	TR Approach	IR Approach
Tax	$L_X(p) - C_T(p)$	$C_{X-T}(p) - L_X(p)$
Transfer	$C_B(p) - L_X(p)$	$C_{X+B}(p) - L_X(p)$

RANGE: to specify a range of p for the search of the first intersection between the two curves. The command also allows to specify the range of the horizontal axis in the drawing of a graph

COMPARING THE PROGRESSIVITY OF TWO TAXES OR TRANSFERS

Let:
- X be gross income;
- $T1$ and $T2$ be two taxes;
- $B1$ and $B2$ be two transfers.

TR progressivity:

$T1$ is more TR-progressive than $T2$ if: $C_{T2}(p) - C_{T1}(p) > 0 \quad \forall p \in]0,1[$

$B1$ is more TR-progressive than $B2$ if: $C_{B1}(p) - C_{B2}(p) > 0 \quad \forall p \in]0,1[$

IR-progressivity:

$T1$ is more IR-progressive than $T2$ if: $C_{X-T1}(p) - C_{X-T2}(p) > 0 \quad \forall p \in]0,1[$

$B1$ is more IR-progressive than $B2$ if: $C_{X+B1}(p) - C_{X+B2}(p) > 0 \quad \forall p \in]0,1[$

To reach this application:

- From the main menu, choose the item: «Redistribution ⇒ Transfer-Tax vs Transfer-Tax".
- In front of the indicators "**Tax (Transfer)**" 1 and 2, specify the two vectors of taxes or transfers.
- Choose the approach to be either TR or IR.
- Choose the different vectors and parameter values.

  Parameters

  p : Percentile

  Vectors

T1 or B1: Tax (transfer) 1

T2 or B2: Tax (transfer) 2

Among the buttons, you find the following commands:

S-GINI: to compute as follows:

	TR Approach	IR Approach
Tax	$IC_{T1}(\rho) - IC_{T2}(\rho)$	$IC_{X-T2}(\rho) - IC_{X-T1}(\rho)$
Transfer	$IC_{B2}(\rho) - IC_{B1}(\rho)$	$IC_{X+B2}(\rho) - IC_{X+B1}(\rho)$

where $IC(\rho)$ is the S-Gini coefficient of concentration and $I(\rho)$ is the S-Gini index of inequality

CROSSING: to seek the first intersection of the two concentration curves. DAD indicates the co-ordinates of that first intersection and their standard deviation if the option of computing with standard deviation is chosen

DIFERENCE: to compute as follows:

	TR Approach	IR Approach
Tax	$C_{T2}(p) - C_{T1}(p)$	$C_{X-T1}(p) - C_{X-T2}(p)$
Transfer	$C_{B1}(p) - C_{B2}(p)$	$C_{X+B1}(p) - C_{X+B2}(p)$

GRAPH: to compute as follows:

	TR Approach	IR Approach
Tax	$C_{T2}(p) - C_{T1}(p)$	$C_{X-T1}(p) - C_{X-T2}(p)$
Transfer	$C_{B1}(p) - C_{B2}(p)$	$C_{X+B1}(p) - C_{X+B2}(p)$

RANGE: "**:** to specify a range of *p* for the search of the first intersection between the two curves. The command also allows to specify the range of the horizontal axis in the drawing of a graph.

COMPARING THE PROGRESSIVITY OF A TRANSFER AND OF A TAX

Let :

- X be gross income;
- T be a tax;
- B a transfer.

TR progressivity:

The transfer B is more TR-progressive than a tax T if: $C_B(p) - L_X(p) > L_X(p) - C_T(p) \quad \forall p \in]0,1[$

IR-progressivity:

The transfer B is more IR-progressive than a tax T if: $C_{X+B}(p) > C_{X-T}(p) \qquad \forall p \in]0,1[$

To reach this application:

- From the main menu, choose the item: "Redistribution ⇒ Transfer vs Tax".
- Choose the approach to be either TR or IR
- Choose the different vectors and parameter values.

Parameters

p: Percentile
ρ: Rho

Vectors

X: Gross income
T: Variable of tax
B: Variable of transfer

Among the buttons, you find the following commands:

S-GINI: to compute as follows:

TR Approach	IR Approach
$2I_X(\rho) - IC_T(\rho) - IC_B(\rho)$	$IC_{X-T}(\rho) - IC_{X+B}(\rho)$

CROSING: to seek the first point at which the progressivity ranking of the tax and transfer is reversed. DAD indicates the co-ordinates of that first reversal and their standard deviation if the option of computing with standard deviation is chosen. These co-ordinates are:

TR Approach	IR Approach
$C_B(p) - L_X(p)$	$C_{X+B}(p)$

DIFERENCE: to seek the first point at which the progressivity ranking of the tax and transfer is reversed. DAD indicates the co-ordinates of that first reversal and their standard deviation if the option of computing with standard deviation is chosen. These co-ordinates are:

TR Approach	IR Approach
$C_T(p) + C_B(p) - 2L_X(p)$	$C_{X+B}(p) - C_{X-T}(p)$

GRAPH: to draw the following curves as a function of p:

TR Approach	IR Approach
$C_T(p) + C_B(p) - 2L_X(p)$	$C_{X+B}(p) - C_{X-T}(p)$

RANGE: to specify the range of the horizontal axis in the drawing of a graph.

HORIZONTAL INEQUITY

A tax or a transfer T causes reranking (and is therefore horizontally inequitable) if:

$$\begin{array}{lll} \textbf{Tax} & : & C_{X-T}(p) - L_{X-T}(p) > 0 \qquad \forall p \in]0,1[\\ \textbf{Transfer} & : & C_{X+T}(p) - L_{X+T}(p) > 0 \qquad \forall p \in]0,1[\end{array}$$

To reach this application:

- From the main menu, choose the item: "Redistribution ⇒ Horizontal inequity".
- Specify if you are using a tax or a transfer.
- Choose the different vectors and parameter values.

Parameters

p : Percentile
ρ : Rho

Vectors

T or B : Tax (transfer)

Among the buttons, you find the following commands:

S-GINI: to compute as follows:

Tax	Transfert
$I_{X-T}(\rho) - IC_{X-T}(\rho)$	$I_{X+B}(\rho) - IC_{X+B}(\rho)$

DIFERENCE: tocompute

Tax	Transfert
$C_{X-T}(p) - L_{X-T}(p)$	$C_{X+B}(p) - L_{X+B}(p)$

GRAPH: to draw the following curves as a function of p:

Tax	Transfert
$C_{X-T}(p) - L_{X-T}(p)$	$C_{X+B}(p) - L_{X+B}(p)$

RANGE: to specify the range of the horizontal axis in the drawing of a graph.

REDISTRIBUTION

A tax or a transfer T redistributes if:

$$\begin{array}{lll} \textbf{Tax} & : & L_{X-T}(p) - L_X(p) > 0 \qquad \forall p \in]0,1[\\ \textbf{Transfer} & : & L_{X+B}(p) - L_X(p) > 0 \qquad \forall p \in]0,1[\end{array}$$

To reach this application:

- From the main menu, choose the item: "Redistribution ⇒ Redistribution".
- Specify if you are using a tax or a transfer.
- Choose the different vectors and parameter values.

Parameters

p : Percentile
ρ : Rho

Vectors

T or B : Tax (transfer)

Among the buttons, you find the following commands:

S-GINI: to compute as follows:

Tax	Transfert
$I_X(\rho) - I_{X-T}(\rho)$	$I_X(\rho) - I_{X+B}(\rho)$

CROSSING: to seek the first point at which the curves $L_{X-T}(p)$ and $L_X(p)$, or $L_{X+B}(p)$ and $L_X(p)$, cross. DAD indicates the co-ordinates of that first crossing and their standard deviation if the option of computing with standard deviation is chosen

DIFERENCE: To compute

Tax	Transfert
$L_{X-T}(p) - L_X(p)$	$L_{X+B}(p) - L_X(p)$

GRAPH: to draw the following curves as a function of p:

Tax	Transfert
$L_{X-T}(p) - L_X(p)$	$L_{X+B}(p) - L_X(p)$

RANGE: to specify the range of the horizontal axis in the drawing of a graph.

THE COEFFICIENT OF CONCENTRATION

Let a sample contain n joint observations, (y_i, T_i), on a variable y and a variable T. Let observations be ordered in increasing values of y, in such a way that $y_i \leq y_{i+1}$. The S-Gini coefficient of concentration of T for the group k is denoted as $IC_T(k;\rho)$ and defined as:

$$IC_T(k;\rho) = 1 - \frac{\sum_{i=1}^{n} \left[\frac{(V_i)^\rho - (V_{i+1})^\rho}{[V_1]^\rho} \right] T_i}{\mu_T} \text{ where } V_i = \sum_{h=i}^{n} w_h^k .$$

To compute the coefficient of concentration for only one distribution:

- From the main menu, choose the following item: "Redistribution ⇒ Coefficient of concentration".
- Choose the different vectors and parameter values.

Parameters

ρ : Rho

Vectors

T : Variable of interest

y : Ranking variable

Among the buttons, you find the following commands:

COMPUTE: to compute the coefficient of concentration.

GRAPH: to draw the value of the coefficient as a function of the parameter ρ.

HORIZONTAL INEQUITY: DUCLOS, JALBERT & ARAAR

With this application, we can decompose the difference between gross income X, and net income N inequality as follows:

$$\Delta I(\varepsilon,\rho) = I_X - I_N = \underbrace{I_X - I_N^E}_{V} - \underbrace{\left(I_N^P - I_N^E\right)}_{H} - \underbrace{\left(I_N - I_N^P\right)}_{R}$$

Where :

- V : Vertical inequality component
- H : Horizontal inequality component
- R : Reranking inequality component
- I_N^P : is the coefficient of concentration of N when the ranking variable is X

$$I_N^E = 1 - \frac{\tilde{\xi}(\varepsilon,\rho)}{\mu_N} \quad and \quad \tilde{\xi}(\varepsilon,\rho) = \sum_{p=0.01}^{1} \xi(N \mid X = Q(p))/100$$

where $\xi(N \mid X = Q(p))$ is the local Gini-Atkinson social welfare index of net incomes N conditional on gross incomes being at their p-quantiles. To compute this, we use a simulated vector of net incomes generated by using the conditional distribution function F(N|X=Q(p)).

The difference in the cost of inequality can also be decomposed as follows:

$$\Delta C = \underbrace{\mu_F - \mu_N^H}_{V^*} - \underbrace{\left(\mu_N^R - \mu_N^H\right)}_{H^*} - \underbrace{\left(\mu_N - \mu_N^R\right)}_{R^*}$$

where

- $\mu_F = \frac{1-I_N}{1-I_X}\mu_N$
- $\mu_N^R = \frac{1-I_N}{1-I_N^E}\mu_N$
- $\mu_N^H = \gamma * \tilde{\xi} \quad and \quad \gamma = \frac{\mu_N^R}{\mu_N}$

To perform this decomposition, follow these steps:

- From the main menu, choose the following item: "Redistribution ⇒ HI: Duclos, Jalbert & Araar".
- Choose the different vectors and parameter values.

Parameters	
ρ :	Rho
ε :	epsilon
Vectors	
X :	Gross Income
N :	Net Income

HORIZONTAL INEQUITY: DUCLOS & LAMBERT;

See the application HORIZONTAL INEQUITY: DUCLOS, JALBERT & ARAAR
The only difference is the parameter ρ which is then set equal to 1.