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THE GINI COEFFICIENT AND NEGATIVE INCOME: A COMMENT

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IN a paper published recently (1982) Chau-Nan Chen, Tien-Wang Tsaur and Tong-Shieng Rhai (henceforth C.T.R.) proposed a modified expression of the Gini coefficient for the case where some incomes Y_i are negative. Their analysis was limited to the case where there exists a k such that $\sum_{i=1}^{k} Y_i = 0$, although in a footnote C.T.R. proposed an expression for the case where $\sum_{i=1}^{k} Y_i < 0$ and $\sum_{i=1}^{k+1} Y_i > 0$. The latter expression however, is not correct and should be written, using their notations, as

$$G^* = \frac{1+2(A-C)}{1+2A} = \frac{(2/n)\sum_{j=1}^{n} jy_j - \frac{n+1}{n}}{1+(2/n)\sum_{j=1}^{k} jy_j + (1/n)\sum_{j=1}^{k} y_j\left[\frac{\sum_{j=1}^{k} y_j}{y_{k+1}} - (1+2k)\right]}$$

since in such a case

$$A = -\frac{1}{n}\left[\tfrac{1}{2}y_1 + (\tfrac{1}{2}y_2 + y_1) + \ldots + (\tfrac{1}{2}y_k + y_{k-1} + \ldots + y_1)\right] + \frac{1}{2n}\frac{\left(\sum_{j=1}^{k} y_j\right)^2}{y_{k+1}}.$$

Another possible, eventually simpler, solution to this problem is to apply Dalton's principle of population (which the Gini coefficient verifies) as follows.

Define

$$\frac{-\sum_{j=1}^{k} y_j}{\sum_{j=1}^{k+1} y_j - \sum_{j=1}^{k} y_j} = \frac{f}{m}$$

where m and f are (positive) integers and f/m is a fraction without common factors. If a new population, of size $n \times m$, is now defined where m individuals earn each the income Y_i ($i = 1, \ldots, n$), it can be easily proved that in such a case there exists a number k' ($0 < k' = mk + f < n$) such that $\sum_{j=1}^{k'} y_j = 0$, in which case the general expression (9) of C.T.R. can be applied without difficulty.

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REFERENCE

CHAU-NAN CHEN, TIEN-WANG TSAUR and TONG-SHIENG RHAI. "The Gini Coefficient and Negative Income", *Oxford Economic Papers*, XXXIV (November 1982), 473–76.